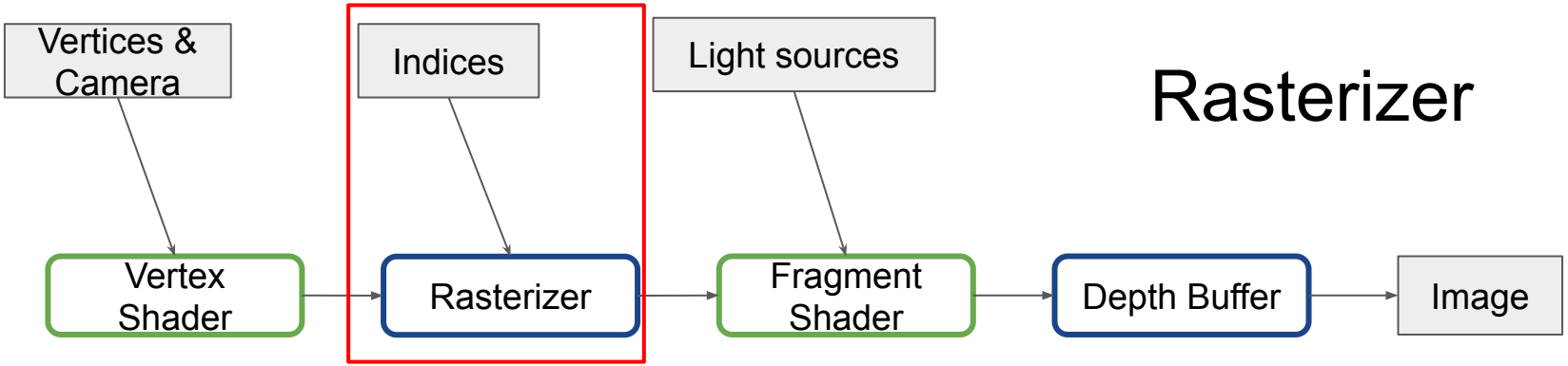
3D Graphics

The rendering pipeline

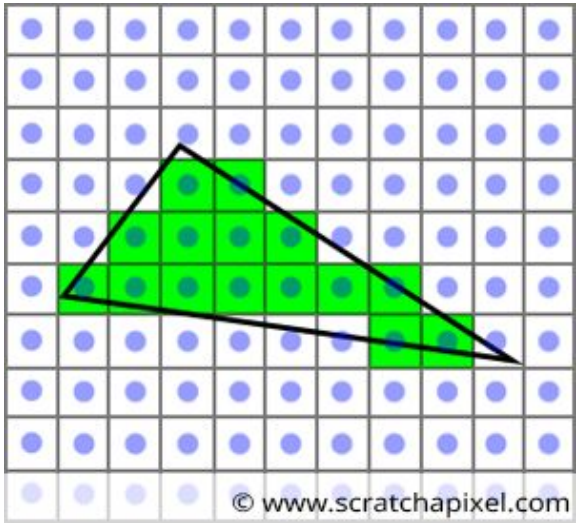
Rasterizer



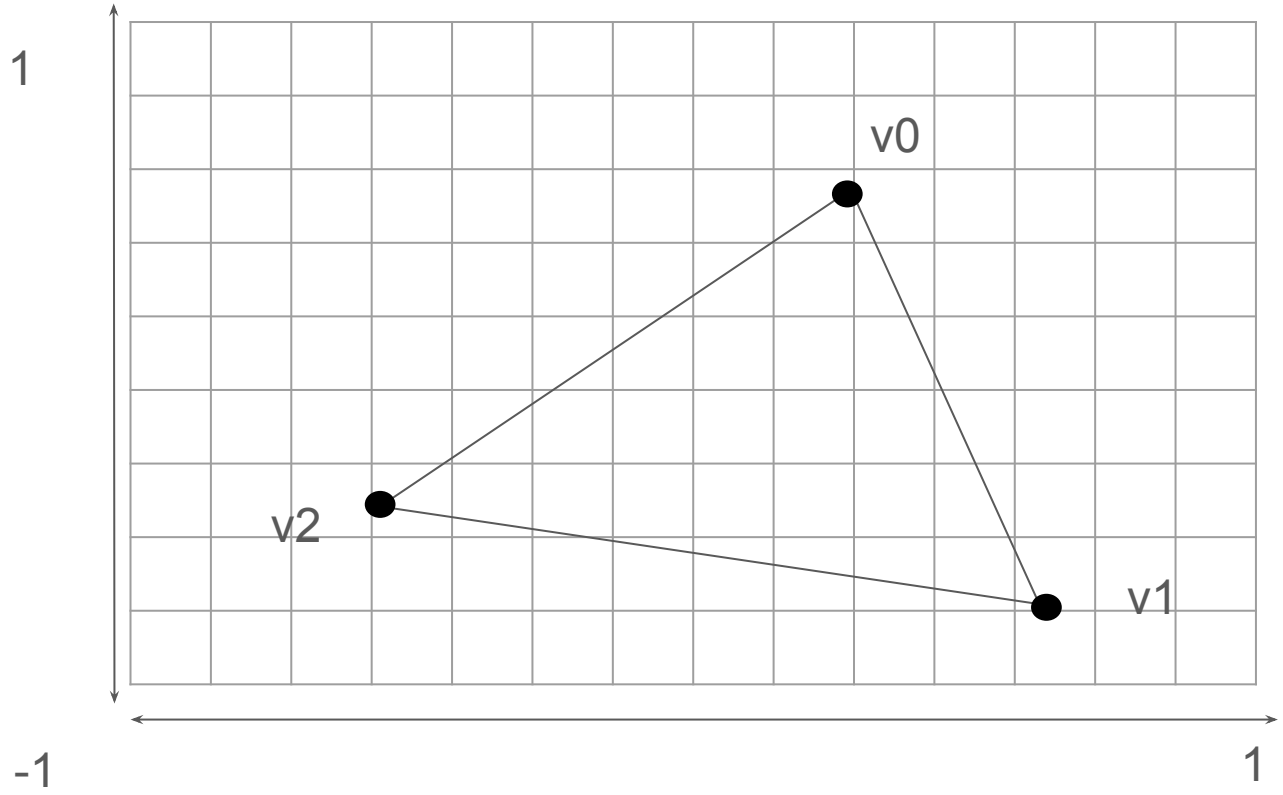
Rasterizer

Find which pixel is inside which triangle

Emit fragments (candidates pixel)



Problem description :

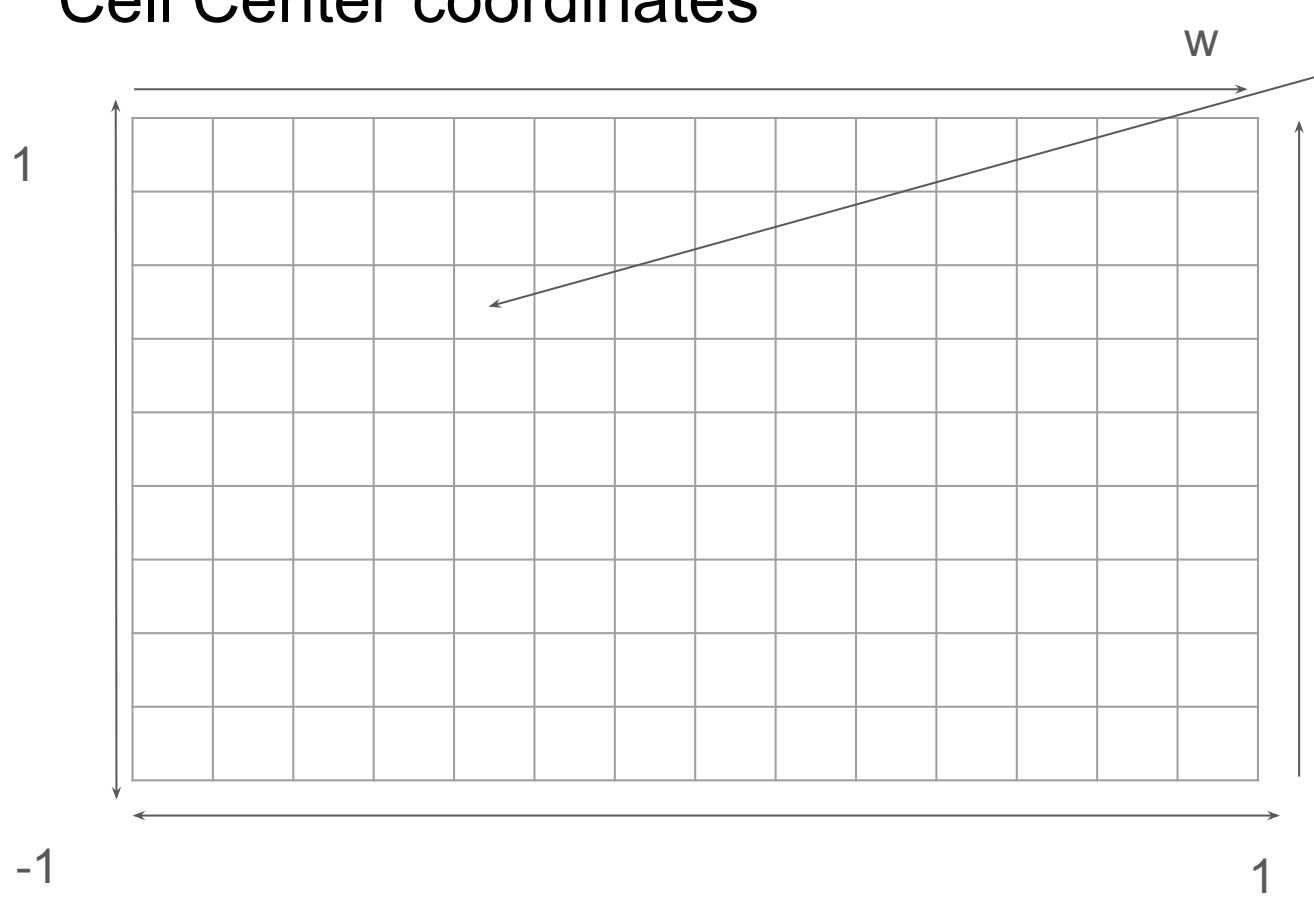


Given :

- A triangle
- A 2D grid of x by y

Which cell **<u>center</u>** of the grid is in the triangle

Cell Center coordinates



$i \in [0,w]$

$j \in [0,h]$

What is the cell center coordinates ?

(1 minute : alone)
(2 minutes : neighbors)
(5 minutes : groups)

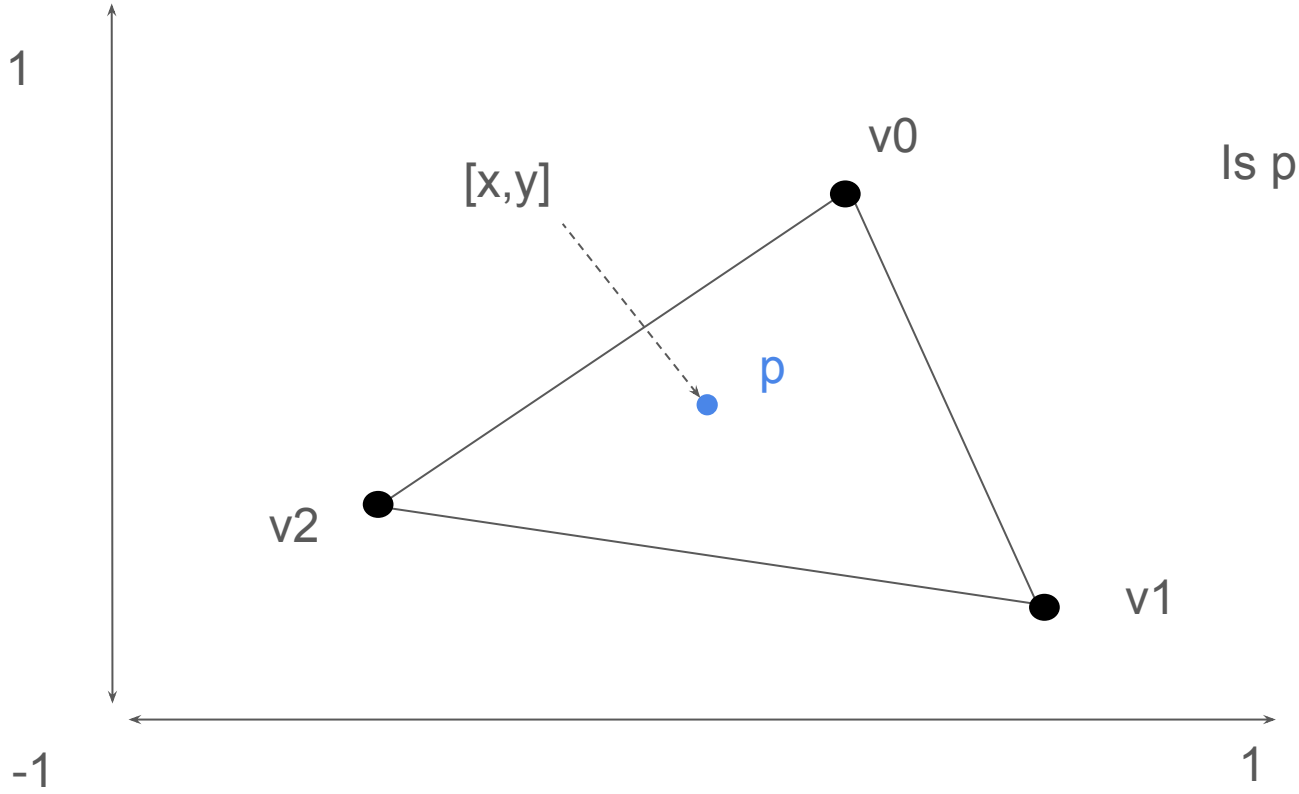
Cell Center coordinates in projected space

$$x = (i + 0.5)/w * 2 - 1$$

$$y = (j + 0.5)/h * 2 - 1$$

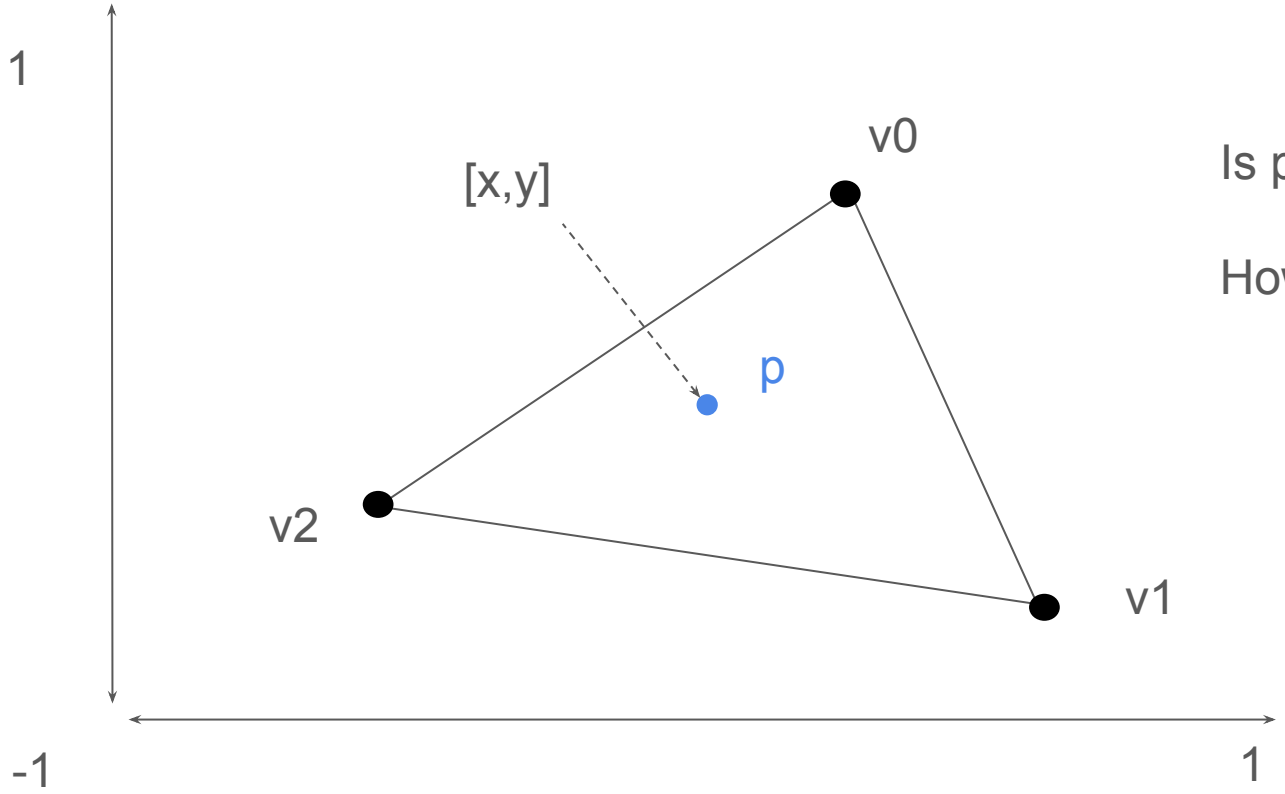
$$\begin{matrix} i + 0.5 \\ j + 0.5 \end{matrix} \longrightarrow \text{Center of Cell i,j}$$

Inside - outside check



Is p inside the triangle ?

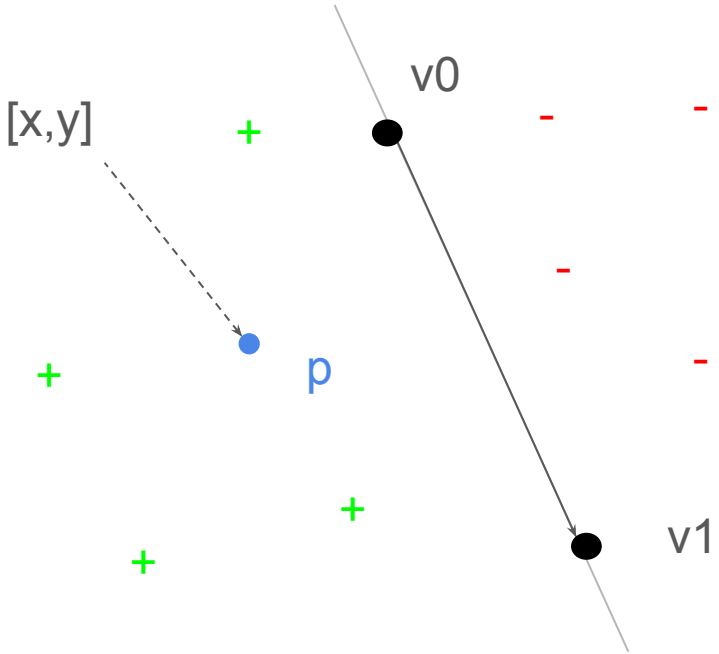
Inside - outside check



Is p inside the triangle ?

How do we know it ?

The edge side function



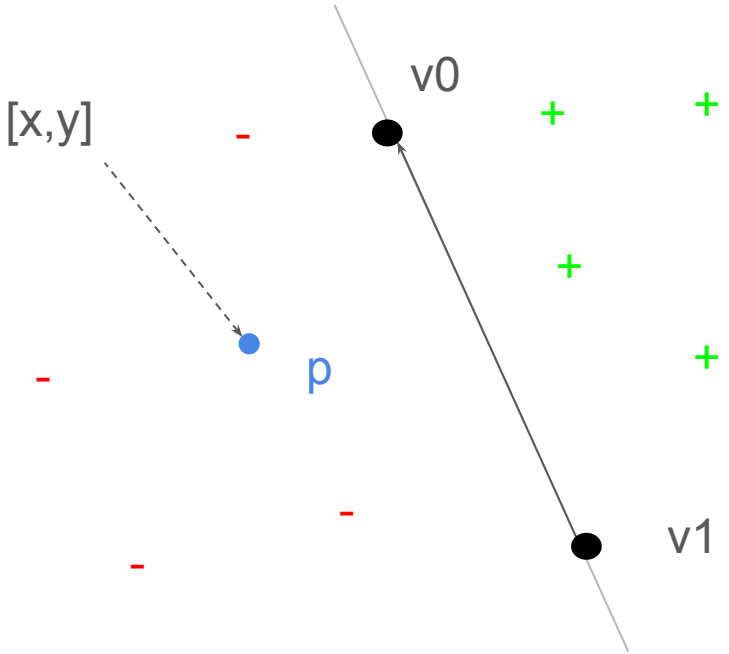
The edge side function return :

A positive value on one side of the edge

A negative value on the other side of the edge

edgeSide(p,v0,v1)

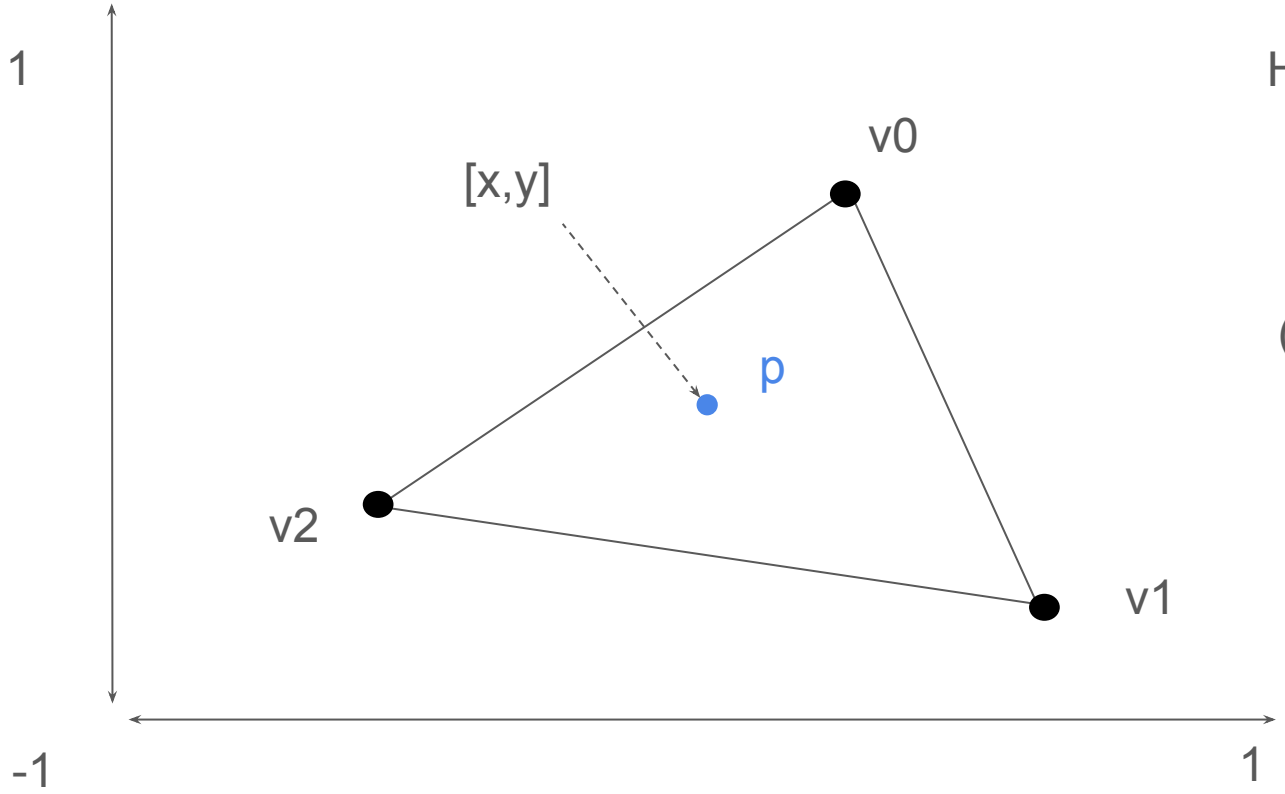
The edge side function : Warning



Reversing the edge orientation change the returned sign

edgeSide(p,v1,v0)

Question



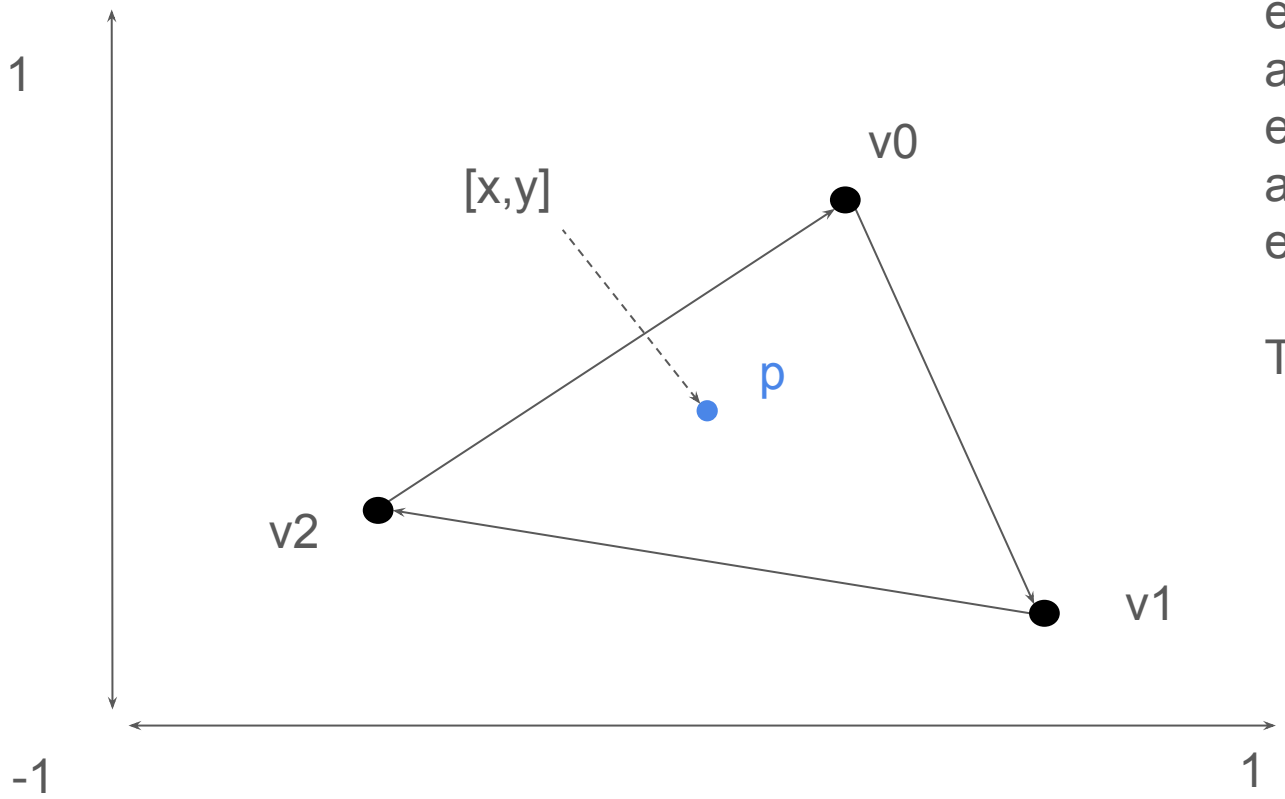
How do we know p is inside the triangle ?

(1 minute alone)

(2 minutes with your neighbors)

(5 minutes with the whole group)

Answer :



If
edgeSide(p,v0,v1) ≥ 0
and
edgeSide(p,v1,v2) ≥ 0
and
edgeSide(p,v2,v0) ≥ 0

Then p is inside

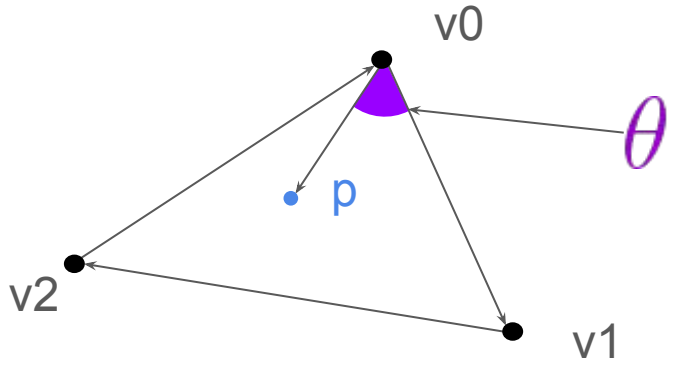
Only works for correctly oriented triangles !

The edge side function

$$edgeSide(p, v0, v1) = (p.x - v0.x)(v1.y - v0.y) - (p.y - v0.y)(v1.x - v0.x)$$

2D cross product magnitude

$$edgeSide(p, v0, v1) = \|(p - v0) \times (v1 - v0)\| = \|(p - v0)\| \, \|v1 - v0\| \, sin(\theta)$$



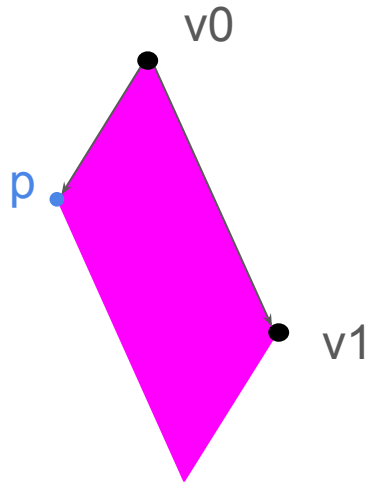
Sign come from the sin function

The edge side function

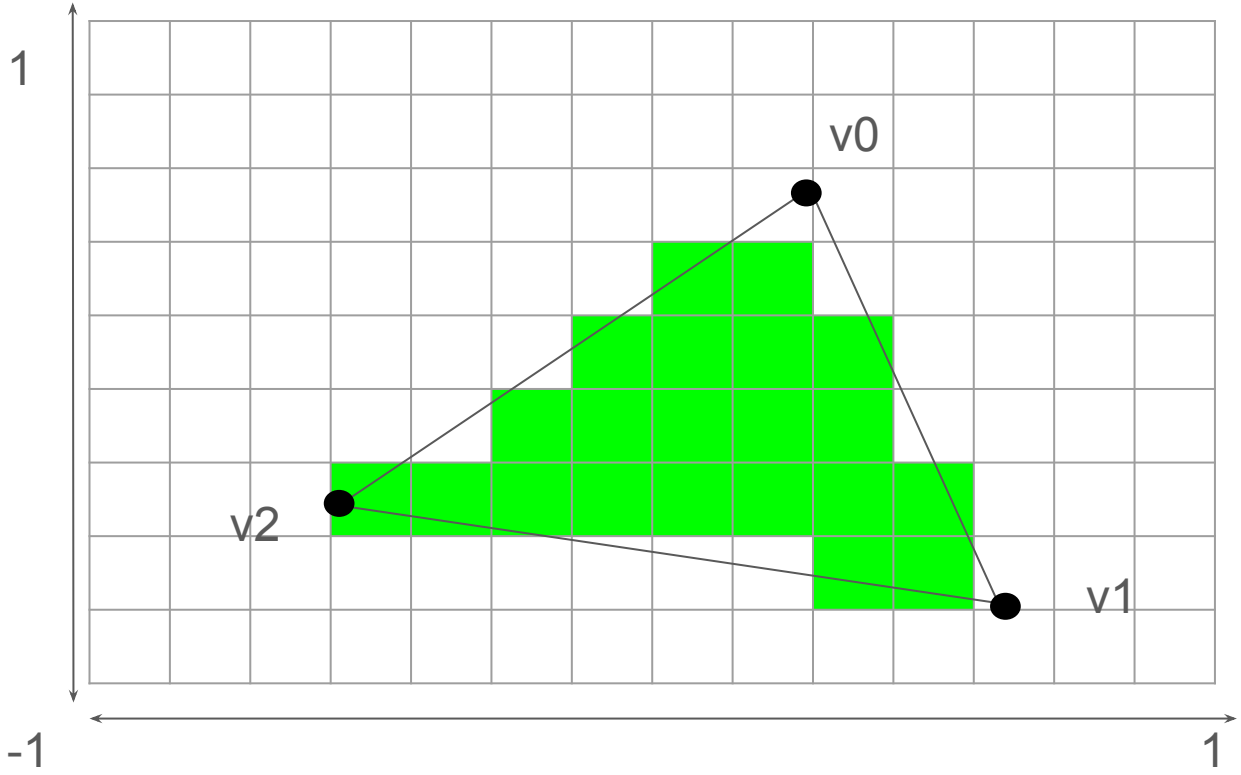
$$edgeSide(p, v0, v1) = (p.x - v0.x)(v1.y - v0.y) - (p.y - v0.y)(v1.x - v0.x)$$

Signed Area

$$edgeSide(p, v0, v1) = Area(parallelograme) = 2 * Area(p, v0, v1)$$

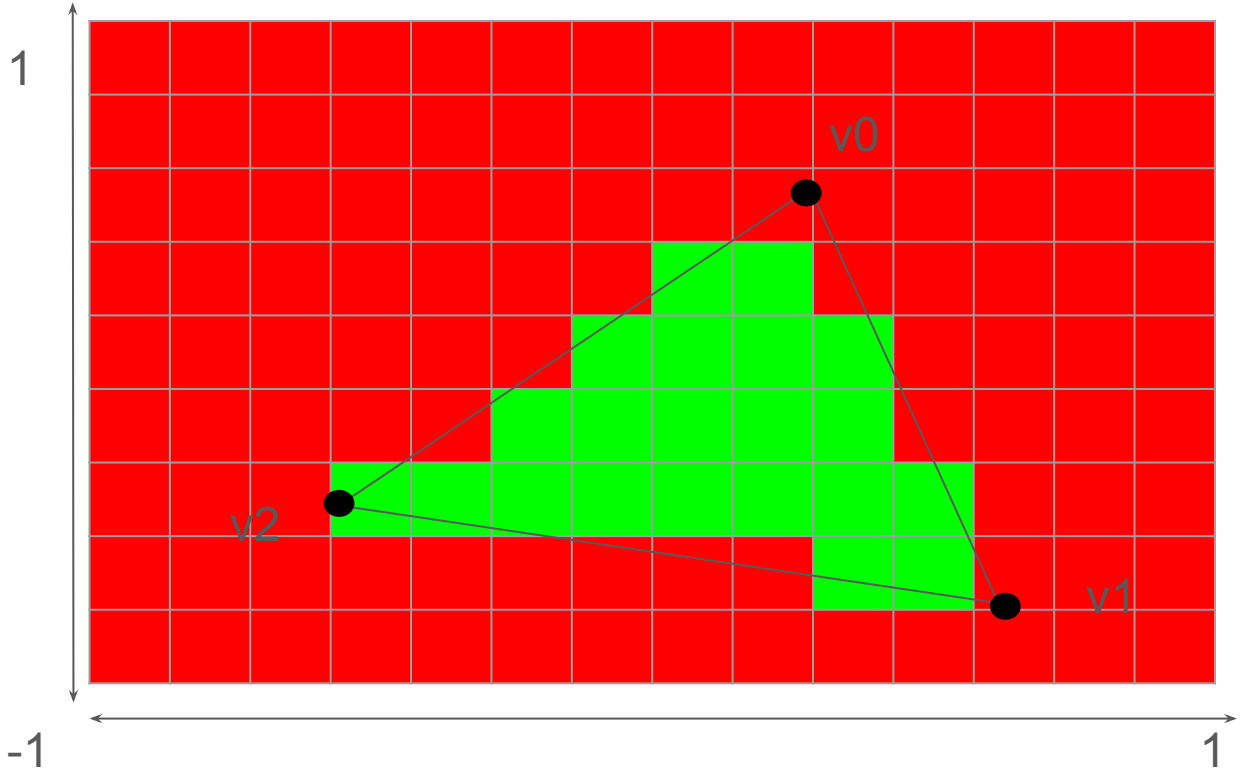


Where are we ?



We can check if a pixel is in a triangle

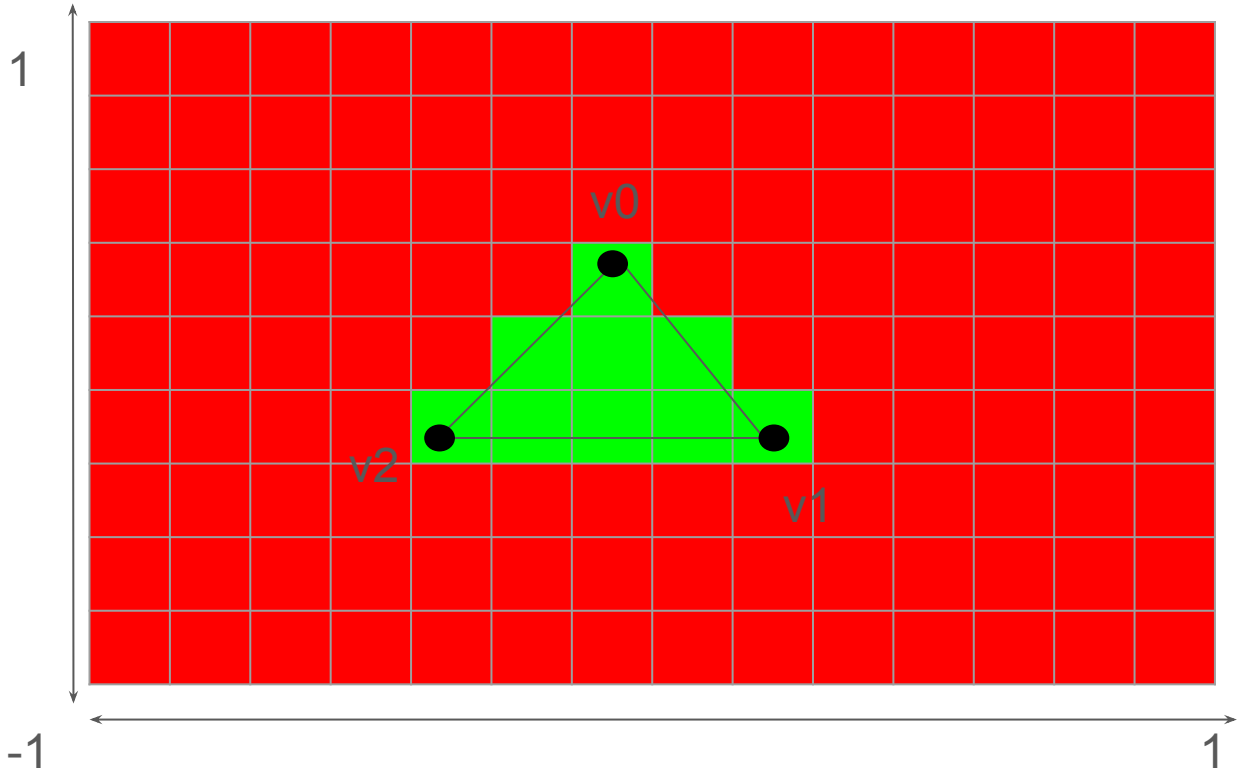
Problem : checking every pixels



Red = pixel tested outside of the triangle

Green = pixel tested inside of the triangle

Problem : checking every pixels



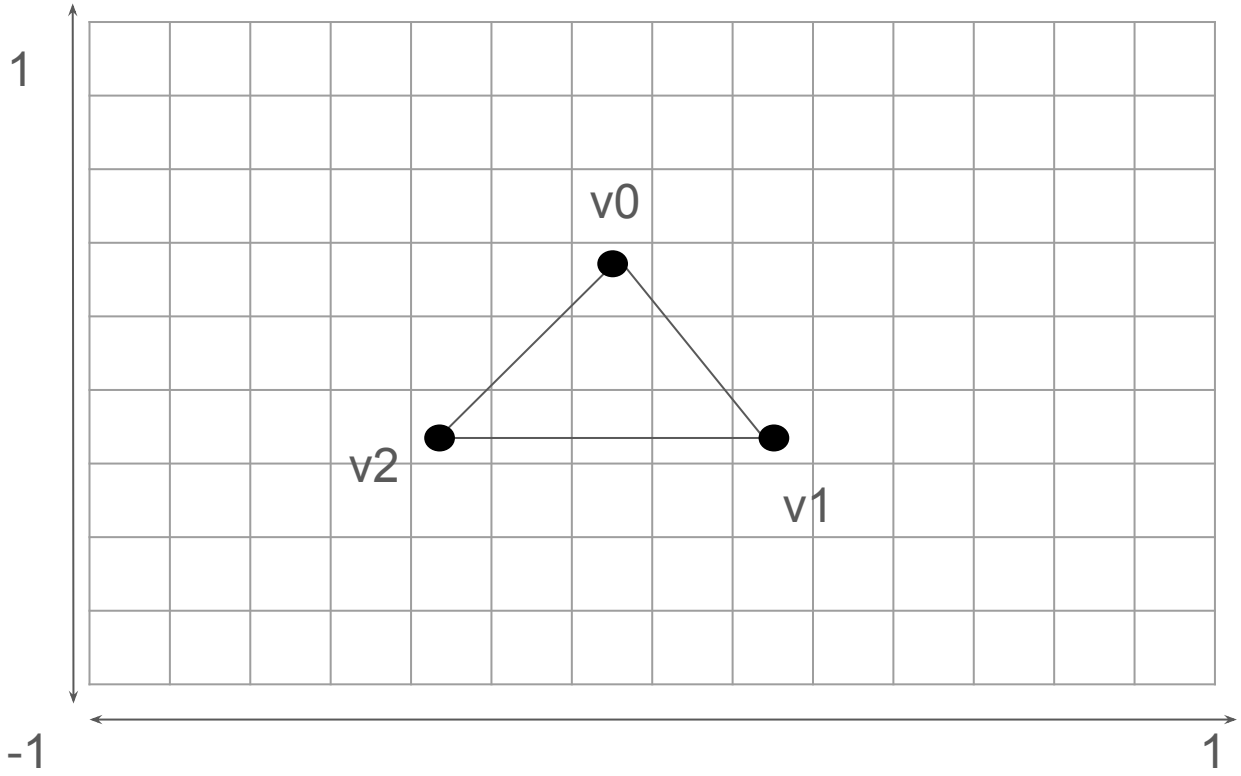
Small triangle:

Red = pixel tested outside of the triangle

Green = pixel tested inside of the triangle

To much inside/outside check !

Question



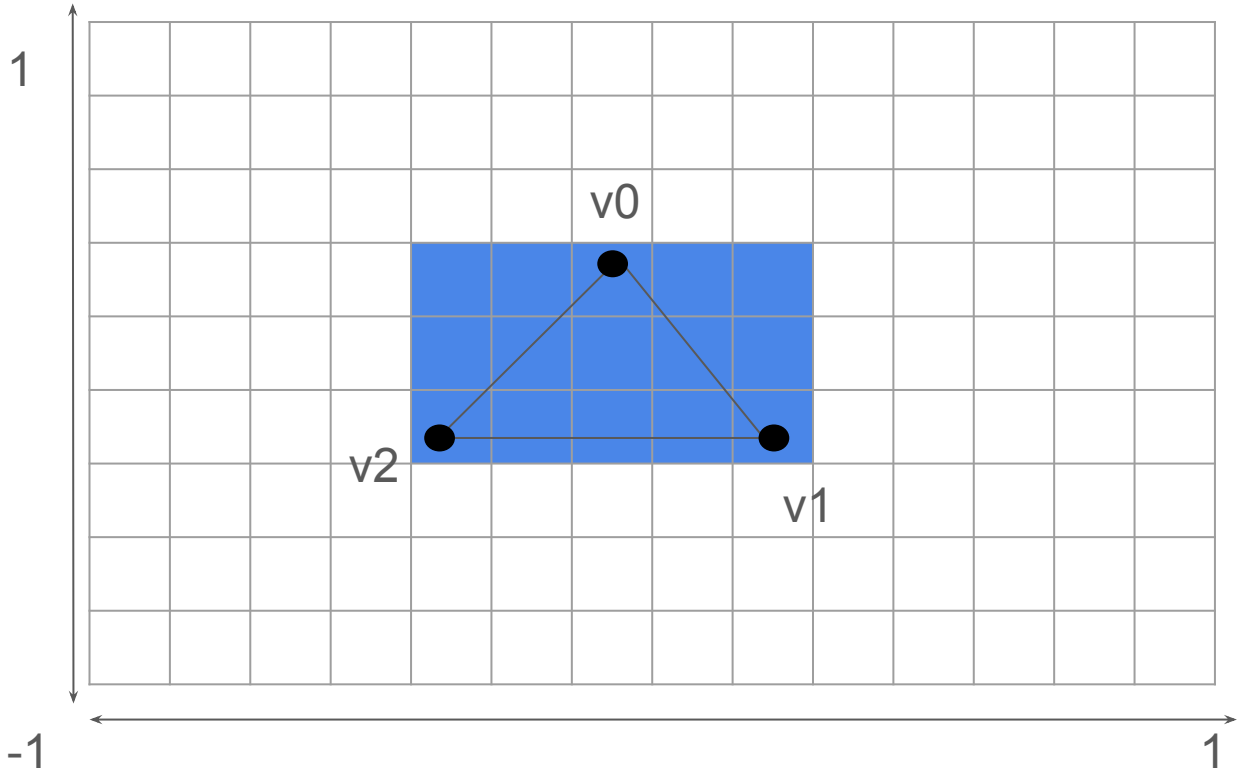
How can we reduce the amount of inside/outside check ?

(1 minute alone)

(2 minutes with your neighbors)

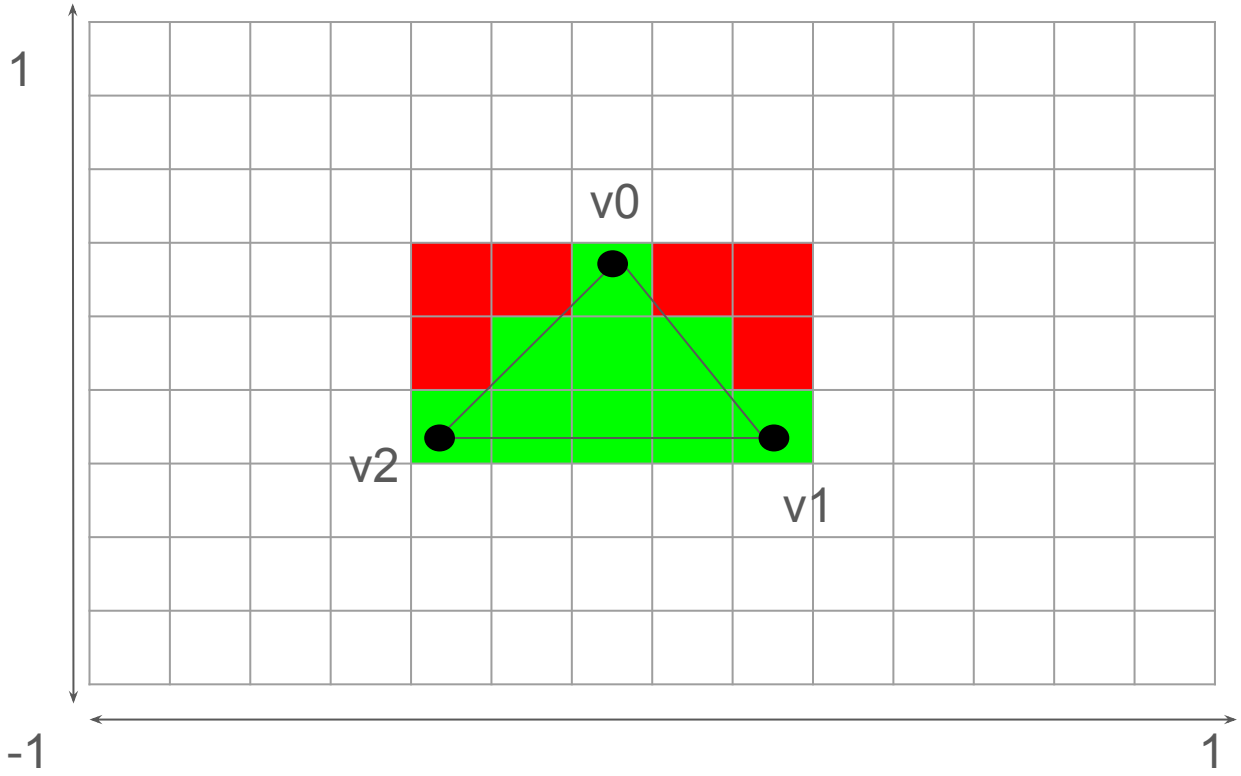
(5 minutes with the whole group)

Answer : Axis Aligned Bounding Box



Only check pixel that are inside the AABBox of the triangle

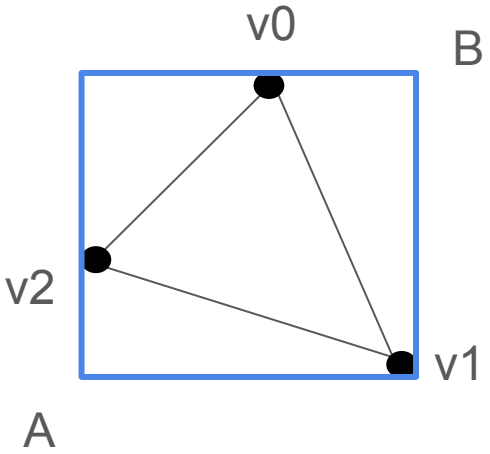
Answer : Axis Aligned Bounding Box



Only check pixel that are inside the AABBox of the triangle

Reduces the number of test on pixel outside of the triangle

Axis Aligned Bounding Box



An AABBox is defined by two points in space.

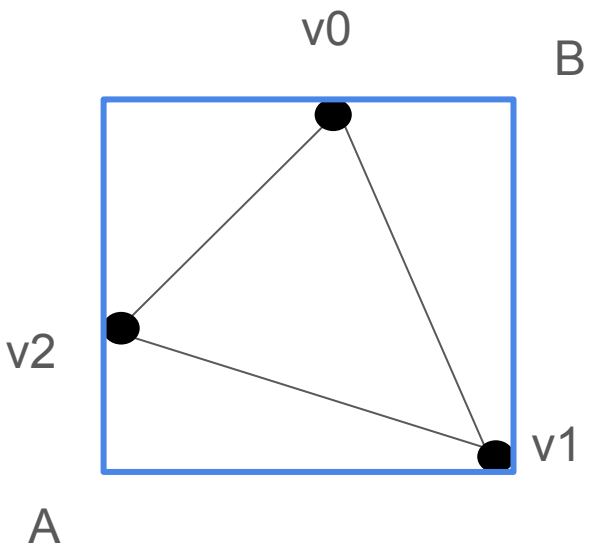
A : the bottom left corner of the box

$$A = \begin{bmatrix} min\,(v0.x, v1.x, v2.x) \\ min\,(v0.y, v1.y, v2.y) \end{bmatrix}$$

B : the top right corner of the box

$$B = \begin{bmatrix} max\,(v0.x, v1.x, v2.x) \\ max\,(v0.y, v1.y, v2.y) \end{bmatrix}$$

Drawing a Triangle

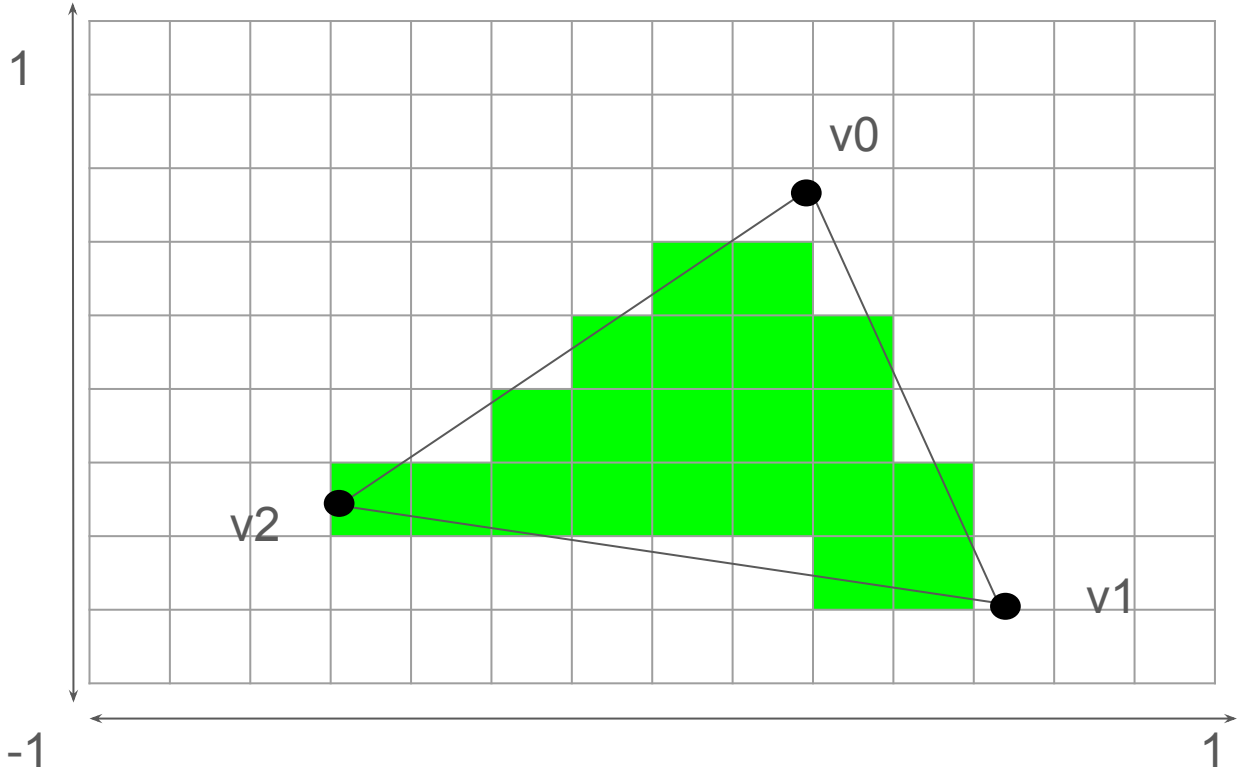


For each pixel p in the AABBox :

If p is inside the triangle :

Emit a fragment*

Fragments



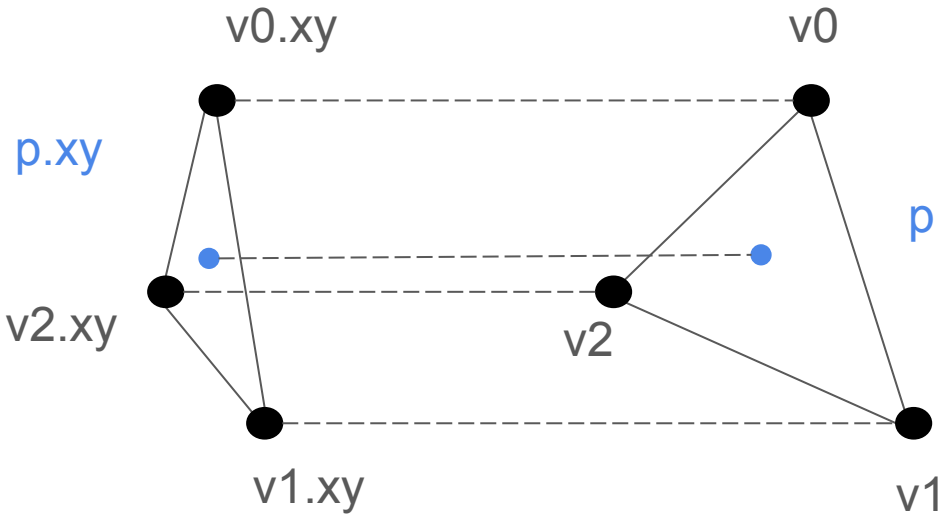
Fragment : Candidate Pixel

Contains :

- Position of the pixel ✓
- Depth of the fragment ?
- Interpolated vertices data*

*Future Lecture

Finding Depth :



How can we find p.z ?

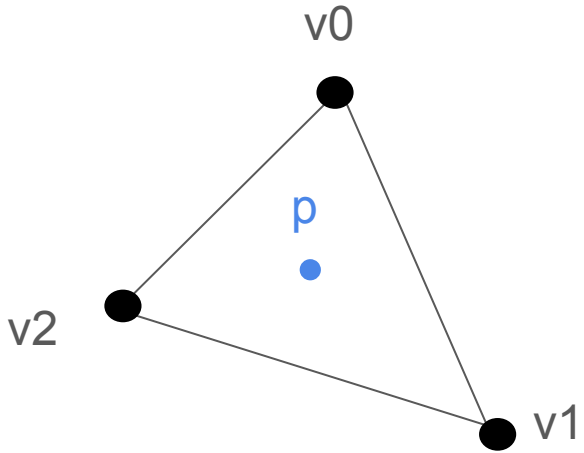
Barycentric coordinates

For every point p in the triangle

$$p = \lambda_0 v0 + \lambda_1 v1 + \lambda_2 v2$$

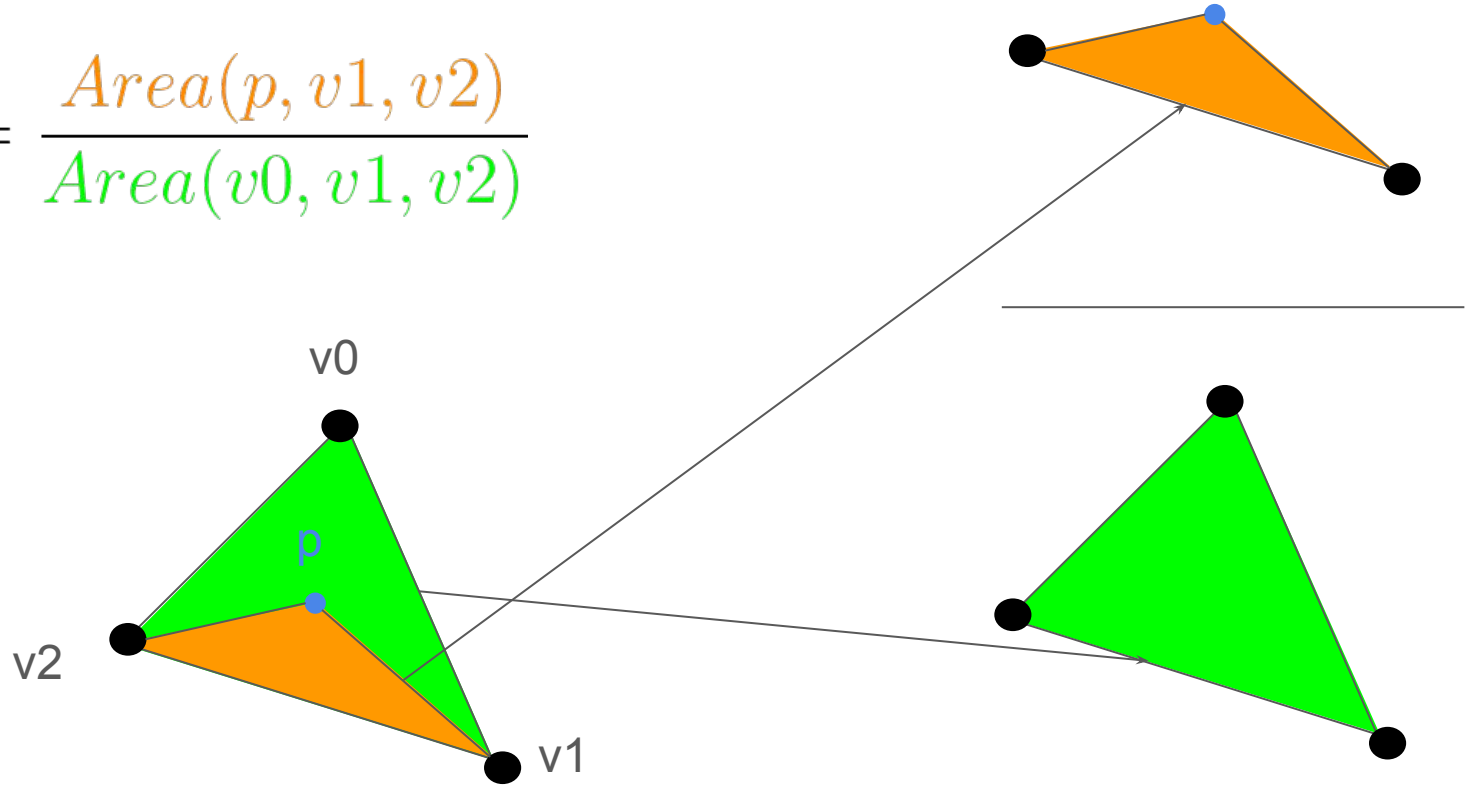
with

$$\lambda_0 + \lambda_1 + \lambda_2 = 1$$



Barycentric coordinates

$$\lambda_0 = \frac{Area(p, v1, v2)}{Area(v0, v1, v2)}$$

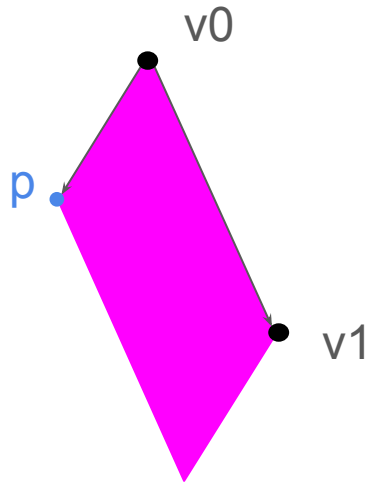


The edge side function

$$edgeSide(p, v0, v1) = (p.x - v0.x)(v1.y - v0.y) - (p.y - v0.y)(v1.x - v0.x)$$

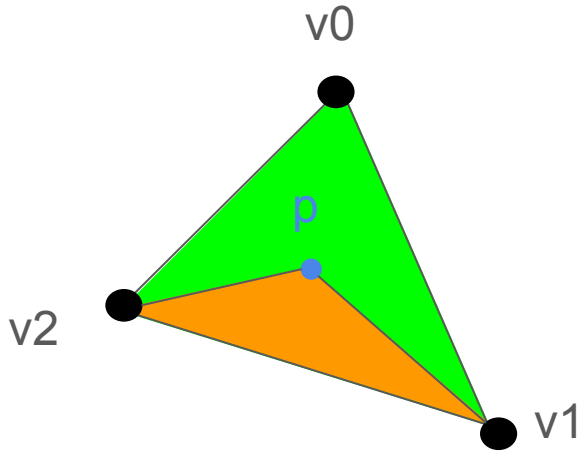
Signed Area

$$edgeSide(p, v0, v1) = Area(parallelograme) = 2 * Area(p, v0, v1)$$



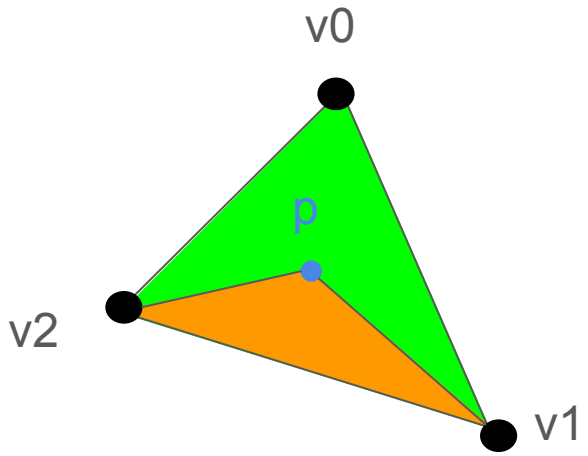
Barycentric coordinates

$$\lambda_0 = \frac{Area(p, v1, v2)}{Area(v0, v1, v2)} = \frac{0.5 \times edgeSide(p, v1, v2)}{0.5 \times edgeSide(v0, v1, v2)}$$



Barycentric coordinates

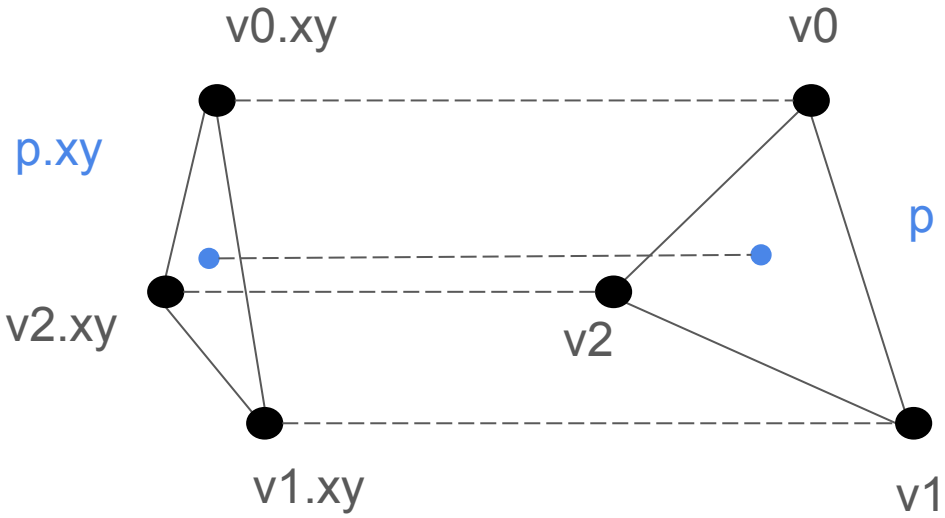
$$\lambda_0 = \frac{Area(p, v1, v2)}{Area(v0, v1, v2)} = \frac{\cancel{0.5 \times}\ edgeSide(p, v1, v2)}{\cancel{0.5 \times}\ edgeSide(v0, v1, v2)}$$



$$\lambda_0 = \frac{edgeSide(p, v1, v2)}{edgeSide(v0, v1, v2)}$$

Finding p.z

$$p.xy = \lambda_0 v0.xy + \lambda_1 v1.xy + \lambda_2 v2.xy$$

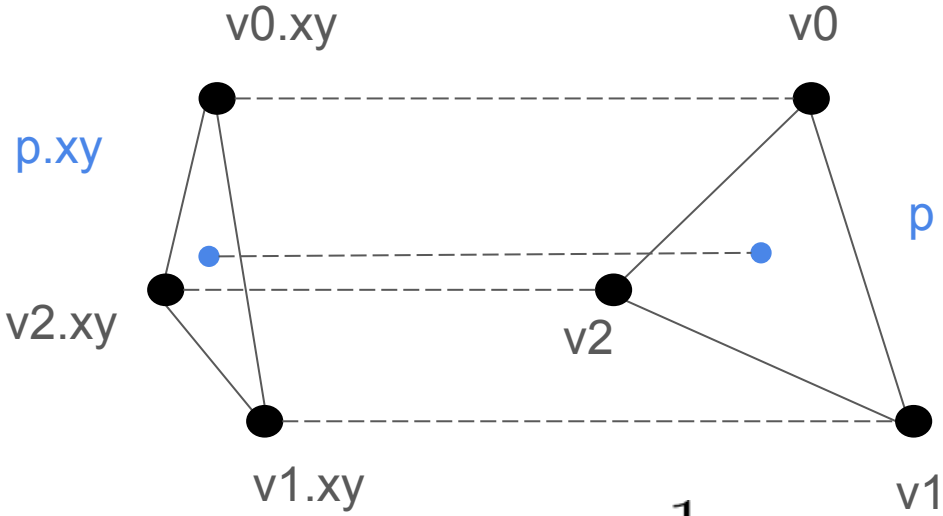


In projected space Z is not linear

$$p.z = \lambda_0 v0.z + \lambda_1 v1.z + \lambda_2 v2.z$$

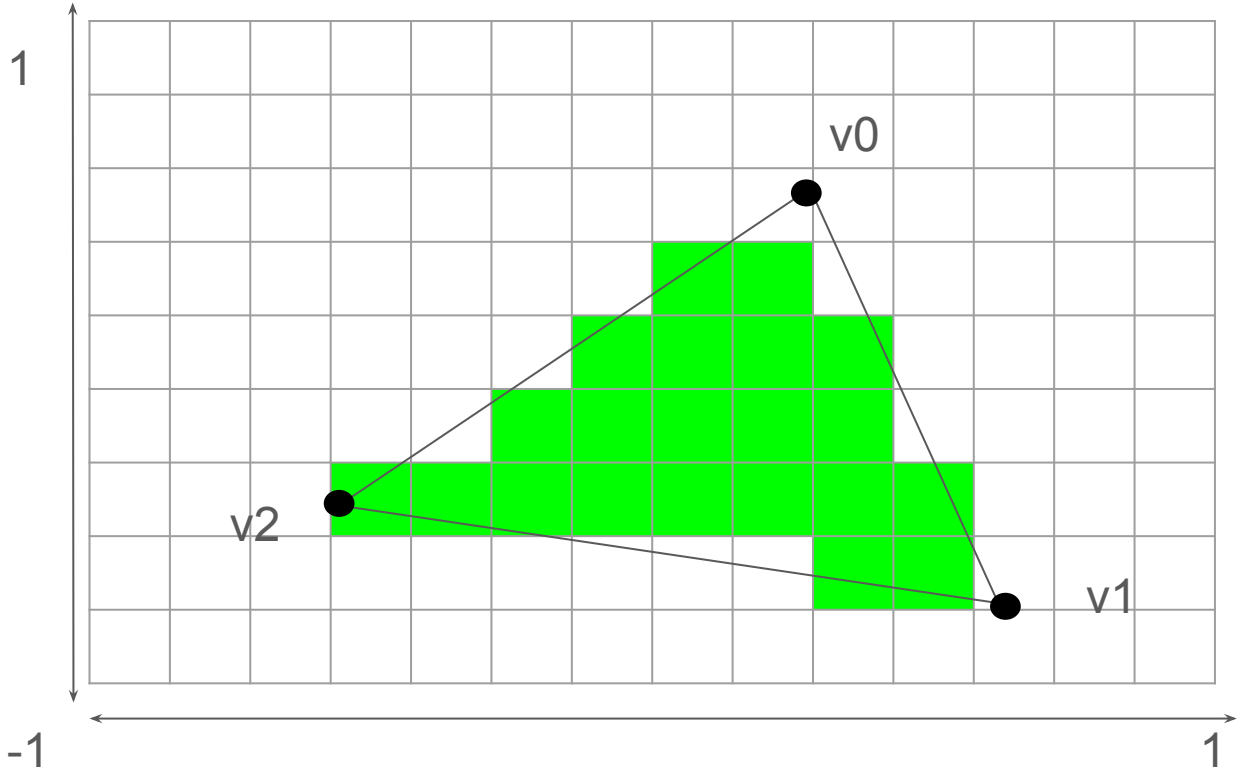
Finding p.z

$$p.xy = \lambda_0 v0.xy + \lambda_1 v1.xy + \lambda_2 v2.xy$$



$$\frac{1}{p.z} = \lambda_0 \frac{1}{v0.z} + \lambda_1 \frac{1}{v1.z} + \lambda_2 \frac{1}{v2.z}$$

Fragments



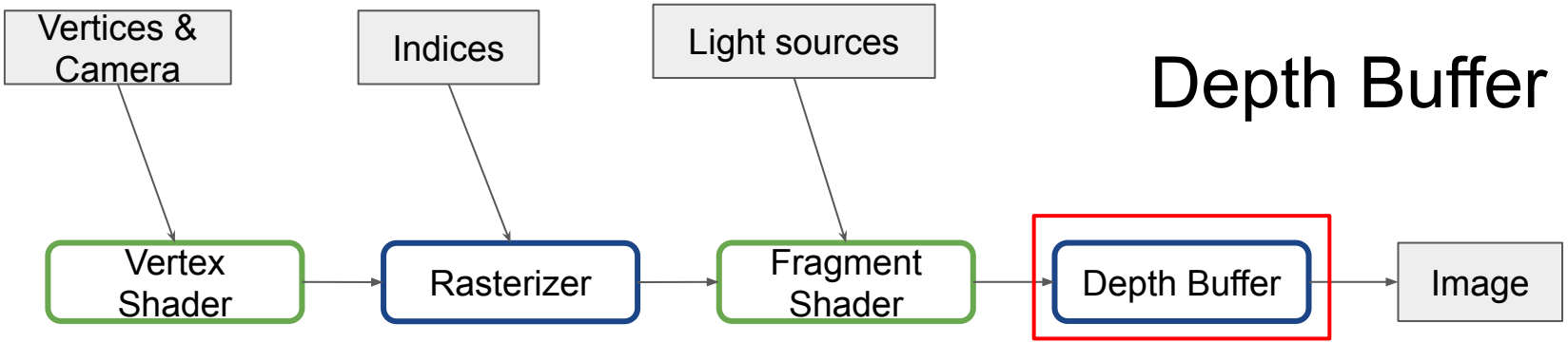
Fragment : Candidate Pixel

Contains :

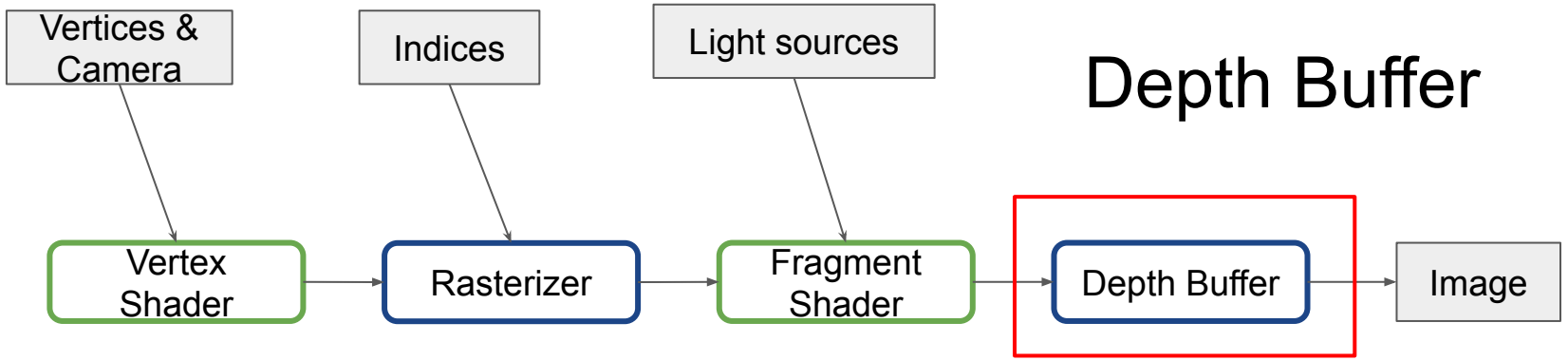
- Position of the pixel ✓
- Depth of the fragment ✓
- Interpolated vertices data*

*Future Lecture

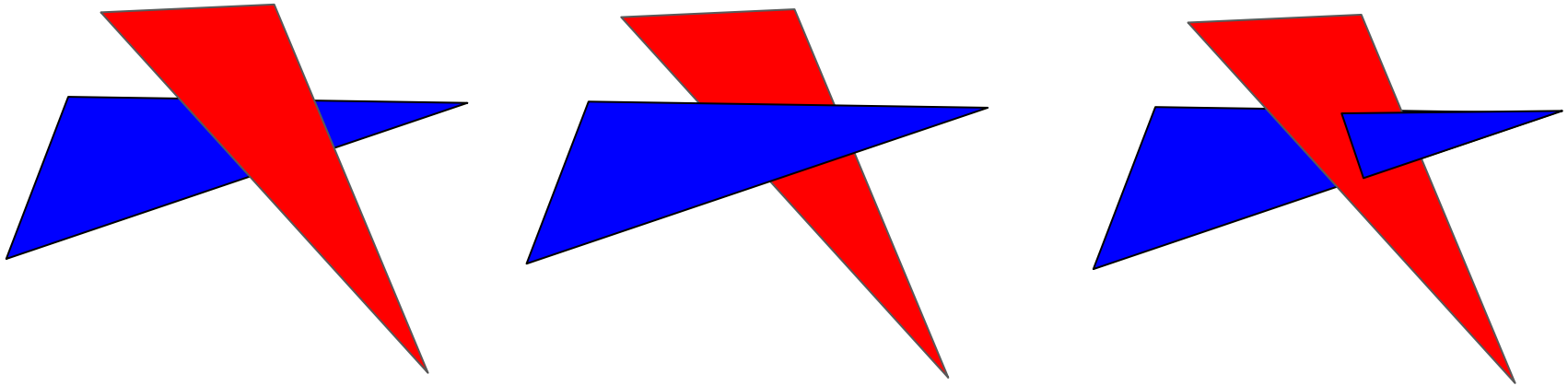
Depth Buffer



Depth Buffer



Choose which Fragment get to become a pixel using a Depth test



Question :

How can we choose which fragment to draw ?

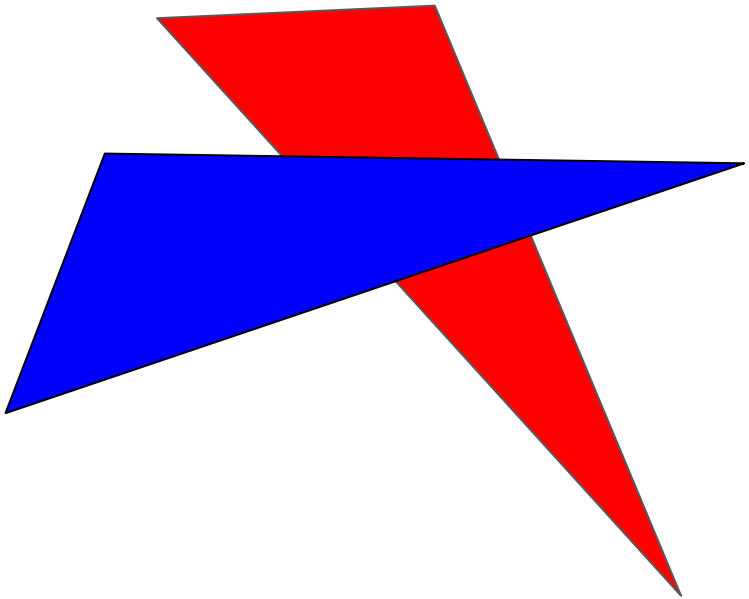
(1 minute alone)

(2 minutes with your neighbors)

(5 minutes with the whole group)

Our use case :

Let’s draw the blue triangle first



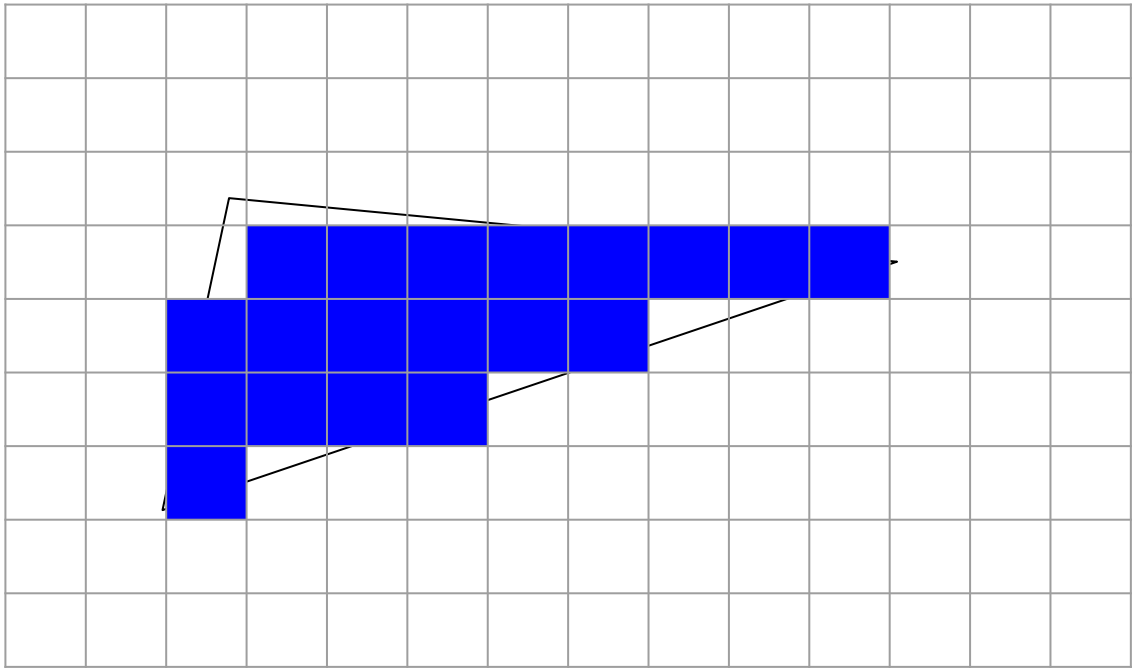
The depth buffer

1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Array with the same size as the image

Initialized with 1.0

The depth buffer

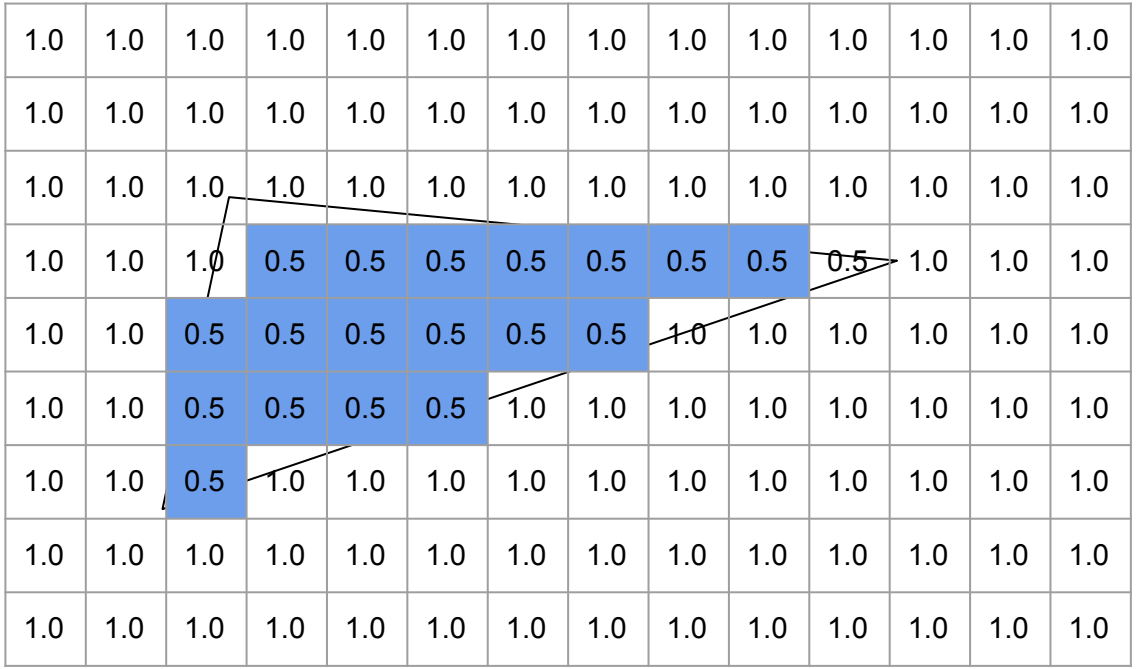


Draw the first triangle

Emit fragment f

If f.z < depthBuffer[f.xy]
  Draw f
  depthBuffer[f.xy] = f.z

The depth buffer



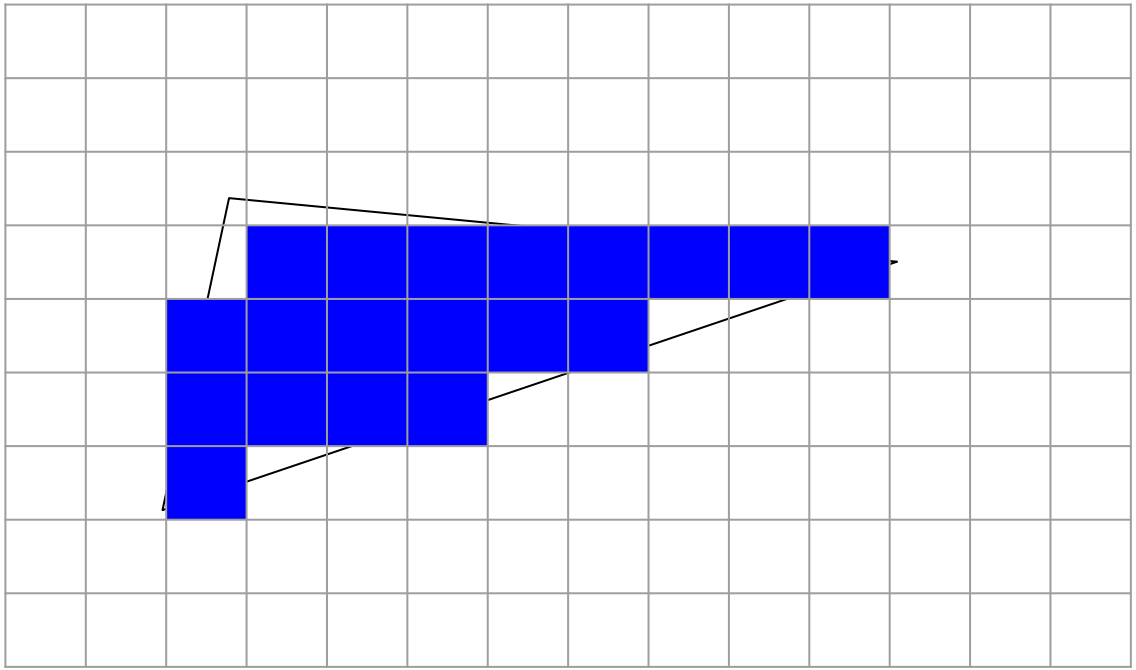
Draw the first triangle

Emit fragment f

If f.z < depthBuffer[f.xy]
  Draw f
  depthBuffer[f.xy] = f.z

triangle1.z = 0.5

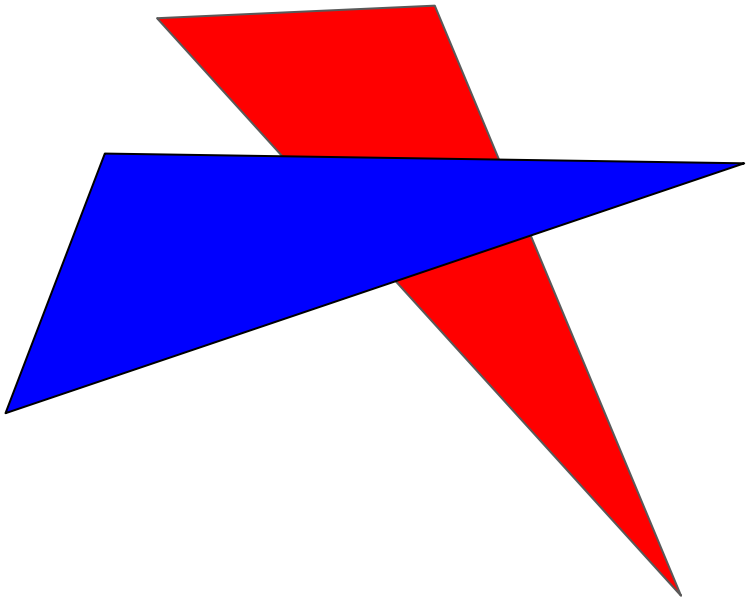
The depth buffer



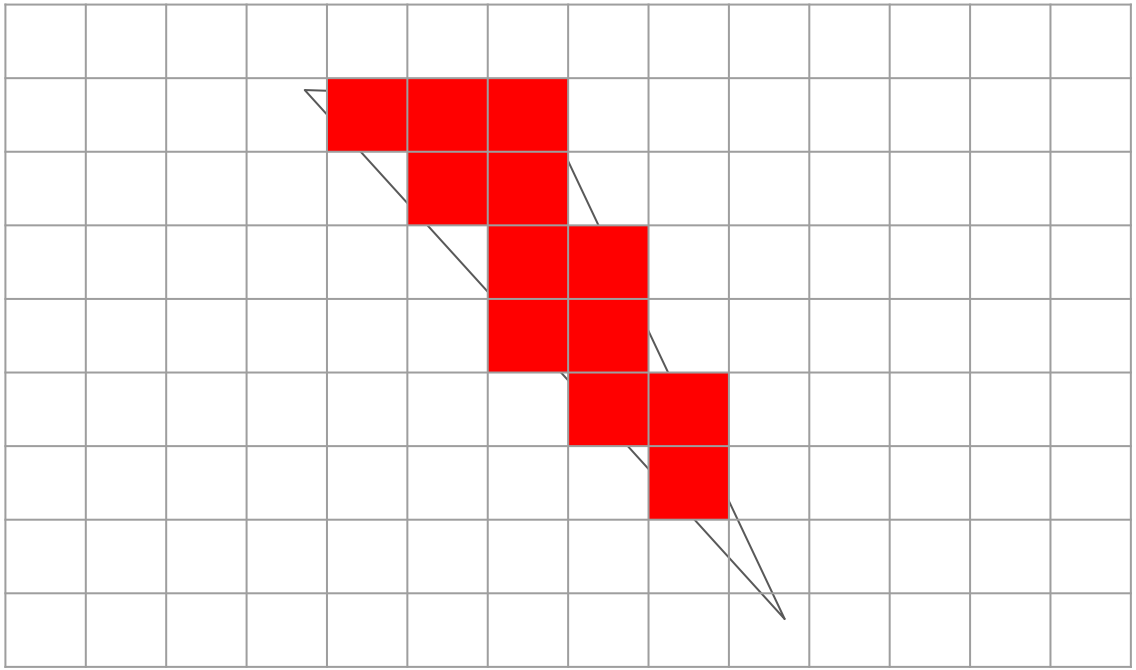
Our image

Our use case :

Now the red triangle



The depth buffer

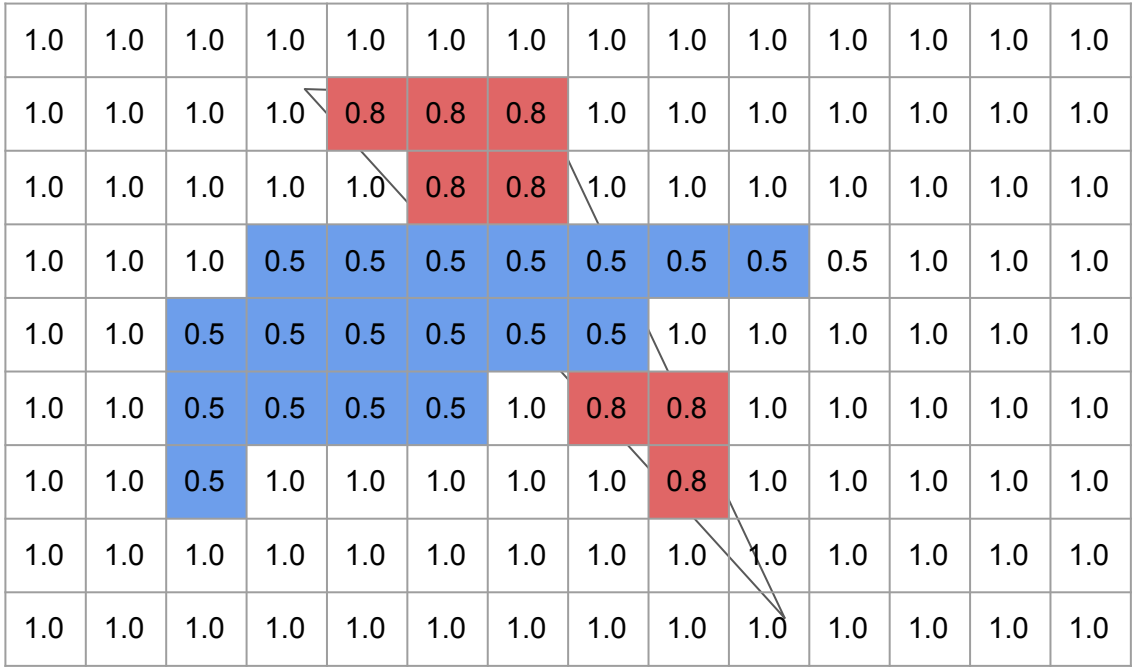


Draw the second triangle

Emit fragment f

If f.z < depthBuffer[f.xy]
  Draw f
  depthBuffer[f.xy] = f.z

The depth buffer



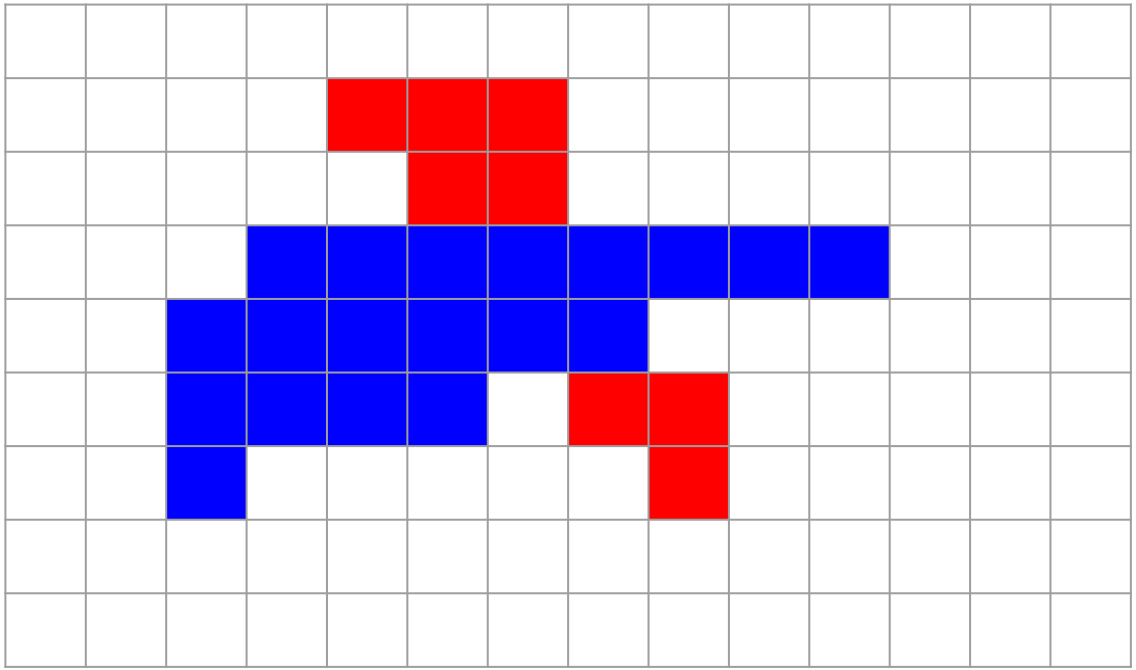
Draw the second triangle

Emit fragment f

If f.z < depthBuffer[f.xy]
  Draw f
  depthBuffer[f.xy] = f.z

triangle2.z = 0.8

The depth buffer



Our image

Depth buffer

The depth Buffer allow us to draw triangle in a whichever order we want regardless of the configuration